Received: *January 27, 1998,* Accepted: *January 29, 1998*

Matching the heavy vector meson theory

Johan Bijnens

Department of Theoretical Physics 2, University of Lund
Sölvegatan 14A, S-22362 Lund, Sweden
E-mail: `bijnens@thep.lu.se`

Peter Gosdzinsky

NORDITA, Blegdamsvej 17
DK-2100 Copenhagen Ø, Denmark

Pere Talavera

Departament de Física i Enginyeria Nuclear
Universitat Politècnica de Catalunya, E-08034 Barcelona, Spain

Abstract: We show how to obtain a "heavy" meson effective lagrangian for the case where the number of heavy particles is not conserved. We apply the method in a simple example at tree level and perform then the reduction for the case of vector mesons in Chiral Perturbation Theory in a manifestly chiral invariant fashion. Some examples showing that "heavy" meson effective theory also reproduces the correct non-analytical terms at the one–loop level are included.

Keywords: 1/N Expansion, Phenomenological Models, Chiral Lagrangians.

Contents

1 Introduction

In a well-known paper[1] Isgur and Wise derived extra consequences of the heavy mass of a particle in a restricted class of processes. Their method was then quickly generalized[2] and extended for use in various sectors of Chiral Perturbation Theory [3, 4]. The general class of processes dealt with in the latter references is with one "heavy" particle going in and moving through the whole process and going out again. Everything else has low momenta compared to the heavy particle. The common point is that the number of "heavy" particles remains constant during the process due to a conserved quantum number. This is not the case in all processes where we expect this type of expansion to be useful. A simple but naive example would be the process in which intermediate state contains particles of even higher masses. Those can be integrated out first and present no theoretical problem. We could, however, study the process $W^+b \to W^+b$ in a theory with a light W-boson. Then we would have to consider the charm quark intermediate state. The charm quark here has a large momentum and the process should be expressible as a power series in $1/m_b$ in an effective lagrangian where the b quark always takes the large momentum.

A similar problem arises when we try to do Chiral Perturbation Theory for vector mesons. In reference [5] was conjectured that a similar formalism should exist there. (See [6] for other applications of this formalism). The main obstacle of dealing with this

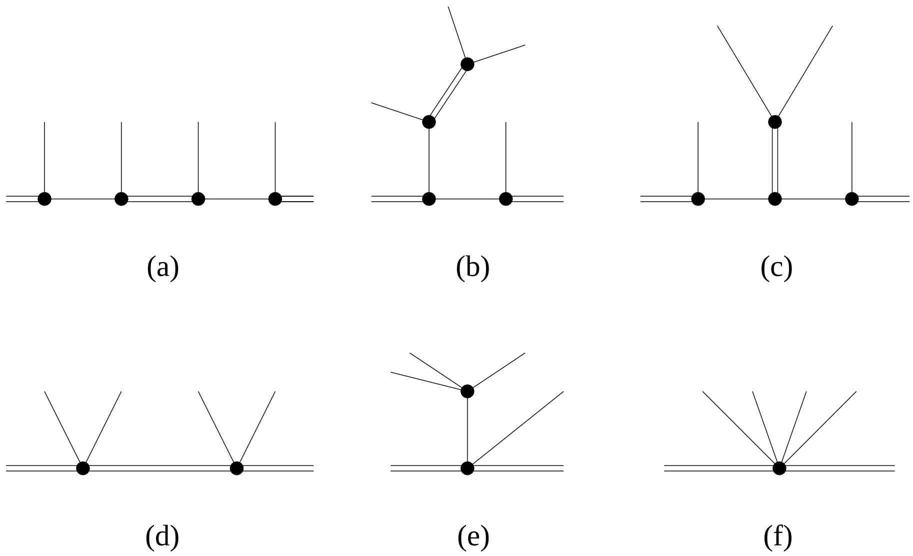


Figure 1: Six point functions in the Simple Model: (a), (b), (c) are in the relativistic formulation and (d), (e) and (f) are their respective counterparts in the HMT. Double lines correspond to the "heavy" field and full lines to the "light" field.

situation is that the methods used in [2] —or e.g. [7] for the extension to other spins — do not take all terms correctly into account in performing the reduction from the full theory to the "heavy" effective one.

In order to estimate the parameters at higher order, one can perform a matching of the relativistic models with the "heavy meson" formulation in a diagrammatic fashion. This diagram by diagram formulation becomes difficult if we want to do the determination of all terms resulting from this reduction. In particular, chiral symmetry relates processes with different number of pions and it would therefore be useful to have a procedure that fully generates the correct terms immediately. For interactions among pions only when integrating out vector mesons this can be done using the equations of motion for the vector meson field[9]. The objective of this paper is to show how this can be done for the "heavy meson" effective theory. The main complication is that we have to correctly treat the diagrams with only light intermediate states, one of which then has to carry high momentum, and at the same time the contributions from the heavy state to subprocesses involving light momenta only. Examples of both are shown in figure 1.

For the "light" and "heavy" states in our theory we thus have to keep two possible momenta regimes. One around the mass-shell of the "light" mode and the other around the mass-shell of the "heavy" mode, see figure 2. In the usual cases only the momentum regime where all particles were close to their mass-shell is kept.[1]

Keeping track of several momentum regions of the particles can in principle be done by introducing several components in the field, each of which has only a low momentum, in that way our Fourier decomposition is:

$$\phi_{\text{full}} = e^{-iMv\cdot x}\tilde{\phi}_v + \tilde{\phi}_0 + e^{iMv\cdot x}\tilde{\phi}_v{}^\dagger\,. \tag{1.1}$$

[1] For the pions close to their mass-shell here means low-momentum.

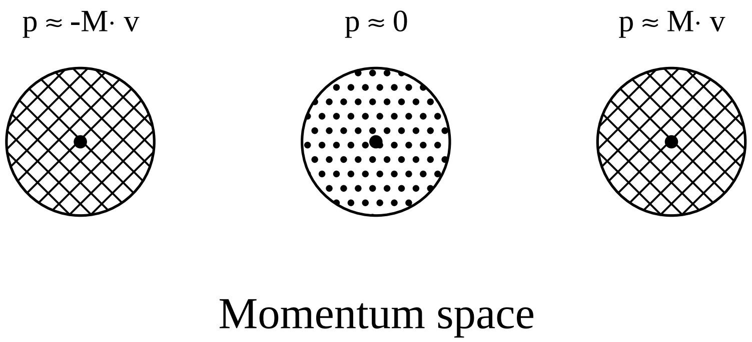


Figure 2: Momenta regions for the Fourier transformations, around the mass-shell of the "light" particle (dotted) and around the mass-shell of the "heavy" particle (crosshatched).

Notice that effective fields are traditionally normalized differently. It corresponds to $\tilde{\phi}_v \to \tilde{\phi}_v/\sqrt{2M}$. This factor should be understood in what follows. The component with the large (small) momentum we will refer to as the high (low) component in order not to confuse with light (heavy) for the mass. The component $\tilde{\phi}_0$ describes the dotted region in figure 2 and $\tilde{\phi}_v, \tilde{\phi}^\dagger$ the crosshatched region.

We then integrate out for the heavy field the component $\tilde{\phi}_0$ and for the light field the component $\tilde{\phi}_v$. This procedure works at tree level since there is always only one line carrying the heavy momentum through the whole diagram. We therefore also only need to keep at most two powers of the high components. The end result is then an effective theory formulated in terms of the high component of the heavy field and the low component of the light field. At any time, terms in the lagrangian which cannot possibly conserve momentum do not contribute since they vanish after the integration over all space.

The approach outlined above will obviously work in simple models. In the chiral models with vertices with any number of fields it becomes rather cumbersome. We will therefore choose a different method. By introducing a "hidden" symmetry we can introduce an extra spurious degree of freedom. The "gauge fixing" of this symmetry consists in choosing an equation for the extra degree of freedom in such a way that the spurious degree of freedom takes care of the far off-shell components of the fields.

We will present the main idea in three stages. First we show it in a simple model where this approach is identical to the one in Eq. (1.1) but shows the procedure in a simple way. Then we show the procedure in a model with an external scalar field coupling to pions in a chirally invariant way. Finally we present the case for vector mesons interacting with pions. Here we rederive and generalize the results of [8].

We also show in a few examples that the "heavy" meson method reproduces the nonanalytic contributions at the one–loop level and present a short discussion of the relevance of the width.

2 A simple model

Consider the following lagrangian with a heavy ϕ field and a light π field:

$$\mathcal{L}_0 = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{1}{2}\partial_\mu\pi\partial^\mu\pi - \frac{1}{2}M^2\phi^2 - \lambda M\phi\pi\pi \ , \tag{2.1}$$

We will be interested in the limit, in which the ϕ mesons gets very heavy, that is, $M \to \infty$. We will consider that λ in this limit is a non-vanishing finite constant.

The "heavy" meson theory (HMT) now looks at processes of the type

$$\phi + n\,\pi \longrightarrow \phi + k\,\pi\,, \tag{2.2}$$

with the ϕ on-shell and the momenta of the pions small compared to the ϕ-mass M. $n\,\pi \longrightarrow k\,\pi$ processes are relevant as well, they occur as subprocesses in (2.2). An important point is to notice that processes such as a decay of a ϕ into two π's, allowed by (2.1), do not lie within the heavy meson theory. The vertex $\lambda M\phi\pi^2$ can, however, generate Green functions that lie within the HMT. In figure 1c, for example, we have combined four of these vertices to construct a six–point function that is described within the HMT. This Green function cannot be generated in the same way in the heavy meson theory, and we will see how the HMT-vertices that accounts for the Green function can be generated.

The naive limit to the heavy meson theory would be to just write

$$\phi = e^{-iMv\cdot x}\tilde{\phi}_v + e^{iMv\cdot x}\tilde{\phi}_v^\dagger \tag{2.3}$$

and restrict $\tilde{\phi}$ and π to momenta much smaller than Mv. Here v is a four-velocity as introduced in [1], $v^2 = 1$. For $\mathcal{L}_0$ this naive procedure would lead to a lagrangian without interaction terms[2]

$$\mathcal{L}_{\text{naive}} = 2Mi\tilde{\phi}_v^\dagger\, v\cdot\partial\tilde{\phi} + 1/2\,\partial_\mu\pi\,\partial^\mu\pi, \tag{2.4}$$

an obviously wrong conclusion.

If we now instead also keep the particular far off-shell regions relevant for the processes of Eq. (2.2) as in Eq. (1.1), we get the relevant interaction terms. To find the equations of motion in the relevant regions for the ϕ and π field we use (1.1) and a similar equation for the π field:

$$\pi = e^{-iMv\cdot x}\tilde{\pi}_v + \tilde{\pi} + e^{iMv\cdot x}\tilde{\pi}_v^\dagger, \tag{2.5}$$

obtaining:

$$\begin{aligned}
0 &= 2iMv\cdot\partial\tilde{\phi}_v - \partial^2\tilde{\phi}_v - 2\lambda M\tilde{\pi}\tilde{\pi}_v \\
0 &= -\partial^2\tilde{\phi}_0 - M^2\phi_0 - \lambda M\tilde{\pi}^2 - 2\lambda M\tilde{\pi}_v\tilde{\pi}_v^\dagger \\
0 &= M^2\tilde{\pi}_v + 2iMv\cdot\partial\tilde{\pi}_v\partial^2\tilde{\pi}_v - 2\lambda M\tilde{\phi}_v - 2\lambda M\tilde{\phi}_0\tilde{\pi}_v \\
0 &= -\partial^2\tilde{\pi} - 2\lambda M\tilde{\phi}_0\tilde{\pi} - 2\lambda M\tilde{\phi}_v^\dagger\tilde{\pi}_v - 2\lambda M\tilde{\phi}_v\tilde{\pi}_v^\dagger\,.
\end{aligned} \tag{2.6}$$

[2]The interaction term always vanishes in this approximation because of momentum conservation.

All the tilded fields have by definition momenta small compared to M, so from (2.6,2.6) it can be seen that $\tilde{\phi}_0$ and $\tilde{\pi}_v$ are suppressed by $1/M$ and can thus be removed recursively in a $1/M$ expansion. Inserting these solutions into the Lagrangian leads to the Lagrangian given below in Eq. (2.13) when the corresponding changes in normalization and notation are done. This method gives the same results as a diagram by diagram matching.

A different way to achieve the same result is introducing first two extra symmetries $R_1 \times R_2$ with the extra fields σ and ψ. The symmetry transformations are with $\alpha_1 \in R_1$ and $\alpha_2 \in R_2$:

$$\phi \to \phi + \alpha_1, \quad \psi \to \psi - \alpha_1, \quad \pi \to \pi + \alpha_2, \quad \text{and} \quad \sigma \to \sigma - \alpha_2\,. \tag{2.7}$$

The lagrangian,

$$\begin{aligned} \mathcal{L}_1 &= \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi + \frac{1}{2}\partial_\mu\Pi\partial^\mu\Pi - \frac{1}{2}M^2\Phi^2 - M\lambda\Phi\Pi\Pi \\ \Phi &= \phi + \psi \\ \Pi &= \pi + \sigma\,, \end{aligned} \tag{2.8}$$

is equivalent to $\mathcal{L}_0$ and has the extra symmetry. We can simply return to $\mathcal{L}_0$ by choosing $\psi = \sigma = 0$. We use the freedom of gauge to have ψ take care of the low part of ϕ and σ of the high part of π. We choose:

$$\begin{aligned} 0 &= -\partial^2\psi - M^2\psi - \lambda M\pi\pi - \lambda M\sigma\sigma \\ 0 &= -\partial^2\sigma - 2\lambda M\phi\pi - 2\lambda M\psi\sigma\,. \end{aligned} \tag{2.9}$$

which in this simple case coincides with the equation of motion for the ϕ and σ fields. Here it should be understood that by momentum conservation, inside the $\sigma\sigma$ term only the "low momentum part" is taken, (i.e. the term behaving like: $2\tilde{\sigma}\tilde{\sigma}^\dagger$, as defined below). This choice now allows us to make the *consistent* set of approximations:

$$\begin{aligned} \pi &= \tilde{\pi} \\ \psi &= \tilde{\psi} \\ \phi &= e^{-iMv\cdot x}\tilde{\phi} + e^{iMv\cdot x}\tilde{\phi}^\dagger \\ \sigma &= e^{-iMv\cdot x}\tilde{\sigma} + e^{iMv\cdot x}\tilde{\sigma}^\dagger\,. \end{aligned} \tag{2.10}$$

This is consistent since the equations of motion are now such that all terms in the equations that contribute to the processes (2.2) are correctly taken care of.[3] The problem before was that, if we used Eq. (2.3) only, there were "driving" terms in the equations that had the wrong momentum[4] that had nowhere to go. The choice of gauge for the spurious fields of (2.9) solves this problem. All the tilded fields in (2.10) are "low momentum fields", that is, their Fourier transforms are only non-vanishing for momenta $p \sim 0$. It is this fact that will allow us to perform a consistent and well defined expansion in $(1/M)$.

[3] This was of course the reason for the choice (2.9).

[4] The other terms can be neglected since at tree level we can never have intermediate lines with momenta far away from either 0 or Mv.

The "heavy" lagrangian should only depend on the fields close to their mass–shell. We therefore integrate out $\tilde{\psi}$, $\tilde{\sigma}$ and $\tilde{\sigma}^\dagger$. At tree level, this can be done by choosing the gauge and replacing the solution in the lagrangian. We have solved (2.9) by performing an expansion in $1/M$. Up to order $\mathcal{O}(1/M^3)$, we find:

$$\begin{aligned}\tilde{\psi} &= -\frac{\lambda}{M}\tilde{\pi}^2 + \frac{\lambda}{M^3}\Box\tilde{\pi}^2 - 8\frac{\lambda^3}{M^3}\tilde{\pi}^2\tilde{\phi}\tilde{\phi}^\dagger \\ \tilde{\sigma} &= 2\frac{\lambda}{M}\tilde{\pi}\tilde{\phi} - 4i\frac{\lambda}{M^2}v\cdot\partial(\tilde{\pi}\tilde{\phi}) + \frac{\lambda}{M^3}\left(2\Box(\tilde{\pi}\tilde{\phi}) - 8(v\cdot\partial)^2(\tilde{\pi}\tilde{\phi}) - 4\lambda^2\tilde{\phi}\tilde{\pi}^3\right) .\end{aligned} \tag{2.11}$$

To go to the nonrelativistic limit, we introduce the correct normalization,

$$\bar{\phi} = \frac{1}{\sqrt{2M}}\tilde{\phi}, \qquad \bar{\phi}^\dagger = \frac{1}{\sqrt{2M}}\tilde{\phi}^\dagger. \tag{2.12}$$

Replacing (2.10), (2.11) and (2.12) in (2.8), and expanding up to order $\mathcal{O}(1/M^2)$, we find:

$$\begin{aligned}\mathcal{L}_1^E &= \frac{1}{2}\partial_\mu\tilde{\pi}^2 + \frac{1}{2M}\partial_\mu\bar{\phi}\partial^\mu\bar{\phi}^\dagger + \frac{i}{2}(\bar{\phi}^\dagger v\cdot\partial\bar{\phi} - \bar{\phi}v\cdot\partial\bar{\phi}^\dagger) + \frac{\lambda^2}{2}\tilde{\pi}^4 - 2\frac{\lambda^2}{M}\tilde{\pi}^2\bar{\phi}\bar{\phi}^\dagger \\ &+ 2i\frac{\lambda^2}{M^2}\left[\bar{\phi}^\dagger\tilde{\pi}v\cdot\partial(\tilde{\pi}\bar{\phi}) - \bar{\phi}\tilde{\pi}v\cdot\partial(\tilde{\pi}\bar{\phi}^\dagger)\right] + \frac{\lambda^2}{M^2}\Big(-\frac{1}{2}\tilde{\pi}^2\Box\tilde{\pi}^2 + \frac{1}{M}\Big\{4\lambda^2\tilde{\pi}^4\bar{\phi}\bar{\phi}^\dagger \\ &- (\tilde{\pi}\bar{\phi}\Box(\tilde{\pi}\bar{\phi}^\dagger) + \tilde{\pi}\bar{\phi}^\dagger\Box(\tilde{\pi}\bar{\phi})) + 4[\tilde{\pi}\bar{\phi}(v\cdot\partial)^2(\tilde{\pi}\bar{\phi}^\dagger) + \tilde{\pi}\bar{\phi}^\dagger(v\cdot\partial)^2(\tilde{\pi}\bar{\phi})]\Big\}\Big) .\end{aligned} \tag{2.13}$$

In (2.13) we have kept terms which apparently are of order $\mathcal{O}(1/M^3)$. The reason is that for Green functions involving $\bar{\phi}\bar{\phi}^\dagger$, an extra factor $2M$ has to be introduced, (2.12). Clearly, we have generated HMT vertices for the Green functions discussed at the beginning of this section. Two four–point vertices are not suppressed in the limit $M \to \infty$. In fact, they are the only non-suppressed Green functions in this limit. The four pion function has been obtained by integrating out the *low* component, $\tilde{\psi}$, of the *heavy* particle, ϕ. This is the same attitude followed for example in [9], where the vector mesons where integrated out to obtain their contributions to the pseudo-scalar interactions at low-energy. The other four–point Green function, $\tilde{\phi}\tilde{\phi}^\dagger\pi\pi$, has been obtained by integrating out the *high* components, $\tilde{\sigma}$ and $\tilde{\sigma}^\dagger$ of the *light* particle, the π. To our knowledge, this procedure was used for the first time in[8], there we used a diagrammatic approach.

We also find a six point function. It is suppressed by a factor $1/M^2$. We have checked that the four and six point functions generated by (2.13) reproduce exactly, up to order $\mathcal{O}(1/M^2)$ the corresponding Green functions of the full relativistic theory, (2.1).

Let us have a closer look at the situation of the six point functions in general, see Fig. 1. The first three diagrams, a, b and c are diagrams in the full theory, (2.1), while diagram d, e and f live in the effective theory.

Cutting the internal heavy meson of diagram a, we obtain two four point functions that lie within the HMT. The HMT can describe these two four–point functions, (2.13). The HMT counterpart for diagram a is diagram d. The situation is very similar for diagram b: here, we again obtain two four point functions that are described within the HMT by cutting one of the internal pion line. The fact that one of these is a four pion function shows why at the beginning of the section we claimed the processes $n\,\pi \longrightarrow k\,\pi$ also have to be taken into account. The HMT counterpart of b is e. The situation is however different for diagram c. Here no HMT diagram can be generated by cutting any of the internal lines. In the HMT, we can not construct its counterpart from "smaller" diagrams. A new vertex, the six point vertex in (2.13) has been generated to account for it. 1PI diagrams in the effective theory can take care of 1PR relativistic diagrams. The HMT counterpart for c is f.

This can be generalized, and if we continue our expansions of $\tilde{\psi}$, $\tilde{\sigma}$, (2.11) and $\mathcal{L}_1^E$, (2.13) to higher orders in $\mathcal{O}(1/M)$, we would generate new eight, ten, and higher order vertices.

The same conclusion can be achieved using the path integral formalism as in the heavy quark effective theory (HQET) [10]. Our problem here is more involved than the one treated there, in the sense that we can have processes where the high component of the flow can be carried by an internal light particle, while in the HQET the high component always correspond to the heavy particle. The main ingredients of the calculation are the inclusion of a low momentum source for the heavy particle (and a high momentum source for the light particle) and the assumption that the determinant can be expanded in the heavy mass. Since the expansion around a certain momentum implicitly implies that only one momentum region is kept in this method it is crucial that first a spurious degree of freedom is introduced in a consistent way as was done here using the hidden symmetry. This particular point together with the study of matching at more than one loop is still under study[13].

3 One loop matching in the simple model

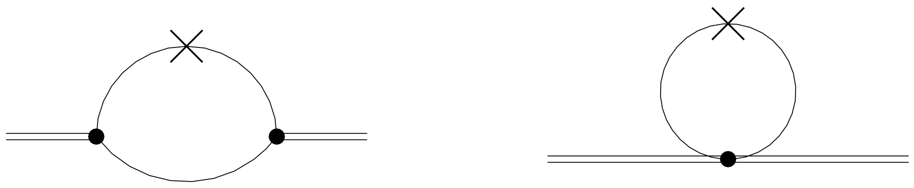
We will now illustrate with a few examples that the effective lagrangian we have constructed in the previous section, which does reproduce correctly the tree level Green functions of the full relativistic theory, also reproduces correctly the non-analytic parts of the Green functions at the one loop level. The first non-trivial results in that model show up in four–point functions. We add a pion mass[5] and a coupling of pions to an external scalar field $S(x)$ to the model. Then we have nontrivial behaviour already in two– and three–point functions. The Lagrangian becomes

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{2} m^2 \pi^2 - S(x)\pi^2 \,. \tag{3.1}$$

The first example will be the one–loop self-energy of the ϕ, at leading order in $1/M$. In the effective theory (HMT), its contribution is given by

$$\Pi_\phi^E = -4i\lambda^2 \frac{m^2}{16\pi^2}\left(-\frac{1}{\epsilon} + \gamma_e - \log 4\pi - 1 + \log\frac{m^2}{\mu^2}\right) + \mathcal{O}(1/M^2), \tag{3.2}$$

[5] For the quantities and to the order considered here the HMT Lagrangian is identical to the one of section 2.



(a) (b)

Figure 3: Scalar form-factor in the Simple Model: (a) relativistic theory (b) effective theory. Double lines correspond to the ϕ field, full lines to the π field, and crosses to insertions of the scalar source $S(x)$.

where we have followed the standard notation and used dimensional regularization with $D = 4 - 2\epsilon$ and γ_e is Euler's constant.

It is also easy to obtain the result in the full relativistic theory:

$$\begin{aligned}\Pi_\phi^F &= 2\lambda^2 M^2 \frac{i}{16\pi^2}\left(\frac{1}{\epsilon} + \log 4\pi - \gamma_e - \log\frac{M^2}{\mu^2}\right) \\ &+ 4\lambda^2 m^2 \frac{i}{16\pi^2}\left(1 - \log\frac{-m^2}{\mu^2} + \log\frac{M^2}{\mu^2}\right) + \mathcal{O}(1/M^2)\ . \end{aligned} \tag{3.3}$$

Let us make a few observations here:

1. The non-analytic terms we are interested in are the logarithms of m^2. It follows from (3.2) and (3.3) that the effective theory does reproduce this. This is the main point.

2. The analytic dependence on m^2 gets a correction at one–loop compared to the tree–level one. This correction is even infinite in the $M \to \infty$ limit. This dependence cannot be obtained from the HMT. We have to leave a term proportional to m^2 for the ϕ mass in the HMT.

3. It is interesting to notice that in the full theory, Π_ϕ^F has an imaginary part, which is absent in the effective theory. This is due to the fact that in the full theory the ϕ has a finite width due to the decay $\phi \to 2\pi$. Since this decay cannot be described within the heavy meson theory, no imaginary part is present in Π_ϕ^E. A small discussion about this fact can be found in section 7.

As a second example we consider the scalar form–factor of the ϕ, with the scalar source $S(x)$ defined above. To leading order in $1/M$, the only diagram that contributes in the effective theory is the one of Fig. 3b. It contributes

$$\frac{8i\lambda^2}{(4\pi)^2}\left[\frac{1}{\epsilon} - \gamma_e + \log(4\pi) - \int_0^1 \log\left(\frac{m^2 - q^2 x(1-x) - i\varepsilon}{\mu^2}\right) dx\right]\ , \tag{3.4}$$

where q is the momentum that flows through the scalar source.

In the full theory, we have to consider only the diagram of figure 3a at this order. Its complete contribution reads

$$I = \frac{8i\lambda^2 M^2}{(4\pi)^2}\int_0^1 \frac{ydxdy}{(-m^2+y(1-y)[Q^2-q^2x]+[xy-(xy)^2]q^2+i\varepsilon)} \,. \tag{3.5}$$

As in the previous case, q is the momentum that enters through the source $S(x)$, and Q is the momentum of the heavy field, ϕ, $Q^2=(Q-q)^2=M^2$. We are only interested in the leading order in $1/M$. The contributions away from $y\approx 0$ and $y\approx 1$ can always be expanded in $1/M^2$, while we expand in $(1-y)$ and y near 1 and 0 respectively:

$$\begin{aligned} I &\approx \frac{8i\lambda^2 M^2}{(4\pi)^2}\int_0^1 dx\left\{\int_{1-\delta}^1 \frac{dy}{(-m^2+(1-y)Q^2+x(1-x)q^2+i\epsilon)}\right. \\ &\quad \left. +\int_\alpha^{1-\delta}\frac{ydy}{y(1-y)Q^2}+\int_0^\alpha \frac{ydy}{-m^2+yQ^2+i\varepsilon}\right\} \\ &= \frac{8i\lambda^2 M^2}{(4\pi)^2}\int_0^1 dx\left(-\frac{1}{M^2}\log\left|\frac{m^2-x(1-x)q^2}{M^2}\right|\right. \\ &\quad \left. -i\,\pi\int_0^\delta dz\,\delta(m^2-x(1-x)q^2-zQ^2)\right)\,, \end{aligned} \tag{3.6}$$

where δ and α are small numbers, but much larger than $\max(m^2,|q^2|)/M^2$. After integrating over y we can again make the same three remarks made above:

1. The non-analytic term in m and q are exactly the same as in the integral of (3.4).

2. The argument of the logarithm is M^2. To obtain μ^2 we have to add a term proportional to $\log(M^2/\mu^2)$. This term requires us to introduce a direct $S(x)\tilde{\phi}\tilde{\phi}^\dagger$ coupling in the HMT to obtain matching.

3. The imaginary part only coincides provided the part corresponding to the ϕ width can be neglected. It is equal to $-\lambda^2M^2/(2\pi)\left\{\theta(1-4m^2/q^2)\sqrt{1-4m^2/q^2}-1\right\}$. The second term is the imaginary part due to the ϕ-width and the first part is the long–distance part present in the effective theory (3.4).

4 A chirally symmetric model

The QCD lagrangian has an approximate chiral symmetry $SU(N_f)_L\times SU(N_f)_R$ for N_f light flavours. This implies the existence of light pseudo-goldstone Bosons, which we will refer to as pions. The method of dealing with these at low energy is Chiral Perturbation Theory[11]. In (2.1) there were only vertices with a limited number of legs. Effective lagrangians with pions tend to have vertices with any number of legs. The method illustrated on the simple example becomes much more powerful here. In this section we will introduce a pion lagrangian coupling to an external scalar source $S(x)$ and we will show how to obtain an effective lagrangian with terms bilinear in $S(x)$ to reproduce all

Green functions of the type $S(-p)S(p+k)\pi^n$ with p a large momentum and k and the π-momenta small. Again we will restrict our discussion to tree level.

Consider the following lagrangian:

$$\mathcal{L}_2 = \frac{F^2}{4}\langle u_\mu u^\mu + S(x)\chi_+\rangle \,. \tag{4.1}$$

Here we used the following notation

$$\begin{aligned}
\chi_\pm &= u_R^\dagger \chi u_L \pm u_L^\dagger \chi^\dagger u_R \\
u_\mu &= i\left(u_R^\dagger(\partial_\mu - ir_\mu)u_R - u_L^\dagger(\partial_\mu - il_\mu)u_L\right) \\
\Gamma_\mu &= \frac{1}{2}\left(u_R^\dagger(\partial_\mu - ir_\mu)u_R + u_L^\dagger(\partial_\mu - il_\mu)u_L\right)
\end{aligned} \tag{4.2}$$

and $\langle\mathcal{C}\rangle$ denotes the trace of $\mathcal{C}$. The external fields χ, l_μ, r_μ are defined in the usual way, see [9]. χ contains the quark masses. All of these are N_f by N_f matrices in flavour space. The matrices u_R and u_L are given in terms of the pions as

$$u_R = u_L^\dagger = u = \exp\left(\frac{i}{F}\pi^a\lambda^a\right) \,. \tag{4.3}$$

The transformations under a chiral transformation $g_L \times g_R \in SU(N_f)_L \times SU(N_f)_R$ are given by

$$u_R \to g_R u_R h_c^\dagger, \quad u_L \to g_L u_L h_c^\dagger, \quad u_\mu \to h_c u_\mu h_c^\dagger \quad \text{and} \quad S(x) \to h_c S(x) h_c^\dagger \,. \tag{4.4}$$

This together with (4.3) determines h_c in terms of the pion fields π^a and g_L, g_R.

We now set

$$S(x) = e^{-iMv\cdot x}\tilde{S}(x) + e^{iMv\cdot x}\tilde{S}(x)^\dagger \,, \tag{4.5}$$

where $\tilde{S}(x)$ only has momenta small compared to Mv and look at Green functions with one insertion of $\tilde{S}(x)$ and one of $\tilde{S}(x)^\dagger$. These have contributions with pions of momenta of order Mv. We treat these pions in the same way as in the previous section by introducing a spurious field that takes its role. The difficulty here is to do it in a manifestly chirally invariant fashion.

This can be done by introducing a "hidden" $SU(N_f)_3$ symmetry that is rather nonlinearly realized. We first introduce three N_f–by–N_f special unitary matrices, $u_\mathcal{R}$, $u_\mathcal{L}$ and $W = w^2$ transforming under $(g_L \times g_R \times h_1 \times h_2) \in SU(N_f)_L \times SU(N_f)_R \times SU(N_f)_1 \times SU(N_f)_2$ as:

$$u_\mathcal{R} \to g_R u_\mathcal{R} h_1^\dagger, \quad u_\mathcal{L} \to g_L u_\mathcal{L} h_2^\dagger, \quad W \to h_1 W h_2^\dagger \tag{4.6}$$

and we define h_3 via

$$w = \sqrt{W} \to h_1 w h_3^\dagger = h_3 w h_2^\dagger \,. \tag{4.7}$$

The other fields are defined via $u_R = u_\mathcal{R} w$ and $u_L = u_\mathcal{L} w^\dagger$ and transform as in (4.4) with h_c replaced by h_3. We remove one of the spurious degrees of freedom by fixing h_1 such that $u_\mathcal{R} = u_\mathcal{L}^\dagger$, which is always possible. The extra symmetry is then the h_2 one with h_1

fixed in this way. The remaining extra degree of freedom is of course also spurious since it can be removed by the choice of h_2. So we have

$$u_R = uw \quad \text{and} \quad u_L = u^\dagger w^\dagger \,. \tag{4.8}$$

Notice that with this w transforms as a pseudoscalar and plays the same role as σ in section 2, and u_R and u_L take the role of Π. We should now use the quantities in (4.1) and (4.2) with (4.8) instead of (4.3). We also use the quantities defined in Eq. (4.2) with a superscript l to indicate those quantities with $w = 1$ (i.e. the *low* components). For the processes we are interested in we expand $w = \exp(i\xi/2)$ up to second order in ξ, there never are more than two lines with a high momentum at any vertex in any diagram.

The equation of motion derived from $\mathcal{L}_2$ is

$$\nabla_\mu u^\mu = \frac{i}{4}\left(\{S(x),\chi_-\} - \frac{2}{N_f}\langle S(x)\chi_-\rangle\right)\,, \tag{4.9}$$

or expanded to second order in ξ:

$$\begin{aligned}
&\nabla^l_\mu u^{l\mu} - (\nabla^l_\mu)^2\xi - \frac{1}{4}[u^l_\mu,[u^{l\mu},\xi]] + \frac{1}{2}[\nabla^l_\mu\xi,[u^{l\mu},\xi]] - \frac{1}{8}[\xi,[\xi,\nabla^l_\mu u^{l\mu}]] = \\
&\frac{i}{4}\left(\{S(x),\chi^l_-\} - \frac{2}{N_f}\langle S(x)\chi^l_-\rangle\right) + \frac{1}{8}\left(\{S(x),\{\xi,\chi^l_+\}\} - \frac{2}{N_f}\langle S(x)\{\xi,\chi^l_+\}\rangle\right) \\
&+\mathcal{O}\left(\xi^3, S(x)\xi^2\right)\,.
\end{aligned} \tag{4.10}$$

Here we have defined $\nabla^l_\mu C = \partial_\mu C + [\Gamma^l_\mu, C]$.

In order to allow a consistent approximation of the quantities only containing u, χ, l_μ, r_μ we now choose the "gauge" for w or ξ such that

$$(\nabla^l_\mu)^2\xi + \frac{1}{4}[u^l_\mu,[u^{l\mu},\xi]] = \frac{-i}{4}\left(\{S(x),\chi^l_-\} - \frac{2}{N_f}\langle S(x)\chi^l_-\rangle\right) + \mathcal{O}\left(\xi^3, S(x)\xi^2\right)\,. \tag{4.11}$$

We can now see that if we restrict $S(x)$ as in Eq. (4.5) and only consider the Green functions mentioned above we obtain a *consistent* approximation scheme using

$$u = \tilde{u} \quad \text{and} \quad \xi = e^{-iMv\cdot x}\tilde{\xi} + e^{iMv\cdot x}\tilde{\xi}^\dagger\,, \tag{4.12}$$

with the tilded quantities with low momenta, since we can use our choice of gauge (4.11) to remove the terms out of the equation of motion for the field u that would have required a high momentum. The equation for $\tilde{\xi}$ starts only at the level $1/M^2$ — this fact will simplify extremely the lagrangian — and can be solved iteratively. Its first two terms are given by:

$$\begin{aligned}
\tilde{\xi} \;=\; & \frac{1}{M^2}\Bigg(\frac{i}{4}(\{\tilde{S}(x),\chi^l_-\} - \frac{2}{N_f}\langle\tilde{S}(x)\chi^l_-\rangle) + \\
& +\frac{1}{2M}\nabla^l_\mu(v^\mu(\{\tilde{S}(x),\chi^l_-\} - \frac{2}{N_f}\langle\tilde{S}(x)\chi^l_-\rangle))\Bigg)
\end{aligned} \tag{4.13}$$

and the resulting effective lagrangian to $\mathcal{O}(1/M^2)$ is

$$\mathcal{L}_2^E = \frac{F^2}{4}\Big[\langle u_\mu^l u^{l\mu}\rangle + \frac{1}{8M^2}\Big(\langle\{\tilde{S}(x),\chi_-^l\}\{\tilde{S}(x)^\dagger,\chi_-^l\}\rangle - \\ -\frac{1}{N_f}\langle\{\tilde{S}(x),\chi_-^l\}\rangle\langle\{\tilde{S}(x)^\dagger,\chi_-^l\}\rangle\Big)\Big] \ . \tag{4.14}$$

The advantage of splitting the fields in such a way is that chiral symmetry remained manifest throughout the whole calculation.

5 Vector mesons

We will now apply our method to the vector meson case. Here the situation is a little bit more involved because of the structure of the heavy meson propagator in the nonrelativistic limit, and we will construct the effective theory in two steps: First, we integrate out the non-relevant degrees of freedom, that is, the low component of the vector mesons and the high components of the pseudoscalars. Then, project the theory on the "orthogonal subspace" (see section IV of [8]), extract the large momentum proportional to M_V from the vectors and construct an effective lagrangian that allows to perform a consistent expansion in $1/M_V$ We again restrict ourselves to the processes of the type:

$$\text{Vector} \ + n\,\pi \quad\longrightarrow\quad \text{Vector} \ + k\,\pi\,. \tag{5.1}$$

There exist several models that describe the interaction of vector mesons with the pseudoscalar Goldstone bosons, see [12] and references therein. We will present our result in terms of model III of that reference (see below for a short discussion about other models). Then the lagrangian reads:

$$\mathcal{L}_3 = \frac{F^2}{4}\langle u_\mu u^\mu\rangle - \frac{1}{4}\langle\overline{V}_{\mu\nu}\overline{V}^{\mu\nu}\rangle + \frac{M^2}{2}\langle(\overline{V}_\mu - \frac{i}{g}\Gamma_\mu)^2\rangle \ , \tag{5.2}$$

with $\overline{V}_{\mu\nu} = \partial_\mu\overline{V} - \partial_\nu\overline{V}_\mu - ig\left[\overline{V}_\mu\,,\,\overline{V}_\nu\right]$. We have disregarded the terms containing quark masses for the present discussion, because they are $\mathcal{O}(1/M)$ suppressed. The pseudoscalar fields transform as in (4.3), and the vector fields transform as:

$$\overline{V}_\mu \to h_c\overline{V}_\mu h_c^\dagger + \frac{i}{g}h_c\partial_\mu h_c^\dagger\,, \tag{5.3}$$

where h_c is defined in (4.4).

We now enlarge the symmetry for the pseudoscalar sector in the same way as we did in the previous section to obtain an extra degree of freedom, $u_R = u_L^\dagger = uw$, and a $\mathcal{R}^4$ hidden symmetry to introduce a spurious vector degree of freedom (corresponding to the ψ field in section 2). We have as a total set of transformations:

$$\begin{aligned} \alpha_\mu \in \mathcal{R}^4 \ &: \ W_\mu \to W_\mu + \alpha_\mu \\ &\quad\ X_\mu \to X_\mu - \alpha_\mu \end{aligned} \tag{5.4}$$

$$\begin{aligned} h_3 \ &: \ W_\mu \to h_3 W_\mu h_3^\dagger \\ &\quad\ X_\mu \to h_3 X_\mu h_3^\dagger + \frac{i}{g}h_3\partial_\mu h_3^\dagger\,. \end{aligned} \tag{5.5}$$

We now use $\overline{V}_\mu = W_\mu + X_\mu$ together with (4.2) and (4.8) in (5.2), with h_3 defined as in section 4. The model is then exactly equivalent to the original model (5.2) setting $X_\mu = 0$ and $\xi = 0$.

Notice that the non-linear term in (5.5) has been chosen inside X_μ, the reason is that later we will choose the $\mathcal{R}^4$ gauge such that this field becomes the "low" momentum component of the vector. It therefore has to take the low momentum nonlinear term. In principle, for the vectors we would have to use the general parametrization (1.1), and we would have to integrate its low component out.

The equation of motion for the pseudoscalar fields reads:

$$\nabla_\mu u^\mu - \frac{iM^2}{2gF^2}[\overline{V}_\mu - \frac{i}{g}\Gamma_\mu, u^\mu] = 0\,, \tag{5.6}$$

and the vector equation of motion:

$$\partial^\mu \overline{V}_{\mu\nu} - ig[\overline{V}^\mu, \overline{V}_{\mu\nu}] + M^2(\overline{V}_\nu - \frac{i}{g}\Gamma_\nu) = 0\,. \tag{5.7}$$

We now expand these equations up to second order in the W and ξ fields.[6] Choosing the gauge fixing conditions such that for the processes (5.1) X_μ only has "low" momentum and ξ only "high" momentum:

$$\begin{aligned}&-(\nabla^l_\mu)^2\xi - \frac{1}{4}[u^l_\mu, [u^{l\mu}, \xi]] \\ &-\frac{iM^2}{2gF^2}\left(\left[W_\mu + \frac{i}{4g}[\xi, u^l_\mu], u^{l\mu}\right] - \left[X_\mu - \frac{i}{g}\Gamma^l_\mu\,,\ \nabla^{l\mu}\xi\right]\right) + \mathcal{O}((\xi, W)^3) = 0\,,\end{aligned} \tag{5.8}$$

for the pseudoscalars and

$$\begin{aligned}&\nabla^{X\mu} X_{\mu\nu} - ig\nabla^{X\mu}[W_\mu, W_\nu] - ig[W_\mu, \nabla^{X\mu} W_\nu - \nabla^X_\nu W^\mu] \\ &+M^2\left(X_\nu - \frac{i}{g}\Gamma^l_\nu - \frac{i}{8g}[\xi, \nabla^l_\nu \xi]\right) + \mathcal{O}((\xi, W)^4) = 0\,,\end{aligned} \tag{5.9}$$

for the vectors. The superscripts l denotes quantities with $w = 1$ as in section 4 and we have defined the following covariant derivative:

$$\nabla^X_\mu \mathcal{A} = \partial_\mu \mathcal{A} - ig[X_\mu, \mathcal{A}]\,. \tag{5.10}$$

This choice now allows us to make the *consistent* set of approximations

$$\sqrt{2M}\; W_\mu = e^{-iMv\cdot x}\tilde{W}_\mu + e^{iMv\cdot x}\tilde{W}^\dagger_\mu \quad \text{and} \quad X_\mu = \tilde{X}_\mu \tag{5.11}$$

together with those of (4.12). Again, tilded symbols means that they are restricted to low momenta.

[6]This is all we need for the processes (5.1) at tree level, when later W and ξ become the "high" momentum components.

Solving iteratively the gauge conditions we find the following solutions for the spurious fields, using $g = M/(2F)$ [12] to count g as $\mathcal{O}(M)$:

$$
\begin{aligned}
\xi &= \frac{i}{2gF^2}[W_\mu, u^{l\mu}] + \ldots \\
\tilde{X}_\mu &= \frac{i}{g}\Gamma^l_\mu + \ldots .
\end{aligned}
\tag{5.12}
$$

Here we only keep the leading terms contributing to (5.1), and set the external sources l_μ and r_μ to zero.

Putting these solutions inside the lagrangian (5.2), after some algebra, we obtain

$$
\begin{aligned}
\mathcal{L}_3^E \; = \; & -\frac{1}{2}\langle \nabla^l_\mu W_\nu \nabla^{l\mu} W^\nu \rangle + \frac{1}{2}\langle \nabla^l_\mu W^\mu \nabla^l_\nu W^\nu \rangle + \frac{F^2}{4}\langle u^l_\mu u^{l\mu} \rangle + \frac{1}{4}\langle [W_\mu, u^{l\mu}]^2 \rangle \\
& -\frac{1}{4}\langle [u^l_\mu, u^l_\nu][W^\mu, W^\nu] \rangle + \frac{M^2}{2}\langle W_\mu W^\mu \rangle + \ldots .
\end{aligned}
\tag{5.13}
$$

The dots in (5.13) denote terms with zero or more than two vectors, as well as terms suppressed in the $1/M$ counting. The term $(1/4)\langle [W_\mu u^{l\mu}]^2 \rangle$ is generated by the high component of the pions, ξ, while the the rest of the terms (excluding the mass term) are generated by the low component of the vectors.

This last step to be taken is to go from (5.13) to an effective theory which allows us to perform a $1/M$ expansion. The presence of terms involving $\partial_\mu W^\mu$ in (5.13) makes this last step a little bit more subtle, and we cannot simply replace (5.11) in (5.13). We will follow the method of section 4 of [8] and introduce a parallel component, $\tilde{W}^\mu_\parallel = v^\mu(v \cdot \tilde{W})$, and a perpendicular component $\tilde{W}^\mu_\perp = \tilde{W}^\mu - v^\mu(v \cdot \tilde{W})$, and integrate the parallel component out, see [8] for details. To leading order in $1/M$, and in the notation of [8] (Eq.(13) of that paper), this leads to

$$
a_2 = -\frac{1}{2M}, \quad a_3 = 0, \quad a_7 = -\frac{1}{4M}, \quad a_8 = 0, \quad a_9 = \frac{1}{4M} . \tag{5.14}
$$

We have checked that these terms can be reproduced *exactly* diagrammatically. The fact that a_3 vanishes in (5.14) might seem surprising, since in (5.13) the term

$$
\frac{1}{2}\langle \nabla^l_\mu W^\mu \nabla^l_\nu W^\nu \rangle \tag{5.15}
$$

is present. To understand $a_3 = 0$, we first notice that (5.15) does not contribute to pion-vector scattering, since for on-shell vectors, $\partial_\mu W^\mu = 0$. For processes such as $2\pi V \to 2\pi V$, the contribution of (5.15) to the diagram of figure 1f does not vanish. To leading order in the $1/M$ expansion, this is cancelled by the contribution of (5.15) to the diagram of figure 1d. Contributions where only one of the vertices of figure 1d is generated by (5.15) are suppressed in $1/M$. We conclude that (5.15) does not contribute to the process $2\pi V \to 2\pi V$ at leading order in $1/M$, and it can be seen that this holds for $n\pi V \to m\pi V$ for any n, m. Eq. (5.14) agrees with [8] after correcting the misprints there.

In this section we have used model III of [12], but similar methods can be applied to the other models. We have checked that all three models predict the same $\langle [W_\mu, u^\mu] \rangle$ term,

JHEP01(1998)014

(coming from the integration out of the high component of the pions) which contributes to the masses in [8], but different $\langle [u_\mu, u_\nu][W^\mu, W^\nu]\rangle$ terms, (integration out of the low components of the vectors) which do not contribute to the masses in [8]. The coefficient we find for this term in model III is 1/4, while in model II we find 1/8. The fact that different models predict different $\mathcal{O}(V^2)$ lagrangians should not be surprising, since $\mathcal{L}_{II}$ and $\mathcal{L}_{III}$ in [12] differ by terms of order $\mathcal{O}(V^2)$.

6 Vectors at one loop

Contributions to the vector masses in the relativistic and the heavy meson formulation are given by the diagrams in figure 3 with the crosses removed. We introduce here a pion mass term, $m = m_\pi$, to have relevant non-analytic contributions and only consider the pion contributions. In the effective formulation we obtain a contribution to the ρ mass-shift

$$\delta M = -\frac{m^4}{32\pi^2 M F^2}\log\left(\frac{m^2}{\mu^2}\right), \tag{6.1}$$

In the relativistic formulation we obtain

$$\delta(M^2) = \frac{M^4}{2g^2F^4}B_{20}(M^2, m^2)\ , \tag{6.2}$$

with

$$\int \frac{d^dp}{i(2\pi)^d}\frac{p_\mu p_\nu}{(p^2-m^2)((p+Q)^2-m^2)} = g_{\mu\nu}B_{20}(Q^2,m^2) + Q_\mu Q_\nu B_{22}(Q^2,m^2)\,. \tag{6.3}$$

We can now check by expanding $B_{20}(M^2, m^2)$ that the first non-analytic dependence on m^2 only appears at order $m^4/M^2\ \log(m^2/M^2)$ and that the coefficients agree if we use the value of g used above. So here we have an indication that the procedure of the heavy meson theory also reproduces properly the non-analytical terms in this case at one loop.

7 Width

The width of the "heavy" particle due to its decays is not included in the HMT. Here we discuss shortly under what circumstances we expect the HMT to be useful given that it cannot easily describe the width. A more exhaustive analysis will be given in [13]. In the tree level processes in the relativistic theory we can describe the width of the heavy particle by using as propagator instead

$$\frac{1}{p^2 - M^2 + iM\Gamma} \approx \frac{1}{M}\frac{1}{2v\cdot k + i\Gamma} \approx \frac{1}{2M}\frac{1}{v\cdot k}\left\{1 - \frac{i\Gamma}{2v\cdot k} + \cdots\right\}\,. \tag{7.1}$$

Here we see that if the typical off-shellness of the "heavy" particle is large compared to its width the latter can be neglected. For vector mesons at tree–level this will always be the case except for the ρ. But even there we are helped by the extra factor of two in the expansion in (7.1).

In the loop diagrams a similar argument will hold if the contributions of the integrals very near to the mass-shell is small compared to the others. Again for vector mesons we expect this to be the case except possibly for the ρ. Even for the ρ, the dependence on the strange quark mass and similar effects come from intermediate states that are far off-shell so those should be reliably estimated in the HMT.

It should also be noticed that the nonanalytic parts of the width are described correctly at the one-loop level considered here. The width at the one-loop level is analytic in the quark masses. Whether the width can be described as well in a HMT way is still an open question and under consideration[13].

8 Conclusions

The problem of reducing a relativistic theory to a “heavy” effective formulation restricted to a particular type of processes has been solved in the case where the number of “heavy” particles is not conserved. The problem of “high” components of light particles and “low” components of heavy particles has been treated in a natural way. We have enlarged the symmetry by a “hidden” symmetry. The spurious degrees of freedom thus introduced, can be chosen by a particular choice of “gauge” for the extra symmetry to play the role of the “high” components of the light particle and of the “low” components of the heavy particle.

The choice of gauge allows then for a simple reduction to the “heavy” effective theory. We explicitly matched all relevant Green functions at tree level, first in a toy model and afterwards in two Chiral models, showing that matching already at this level is rather subtle. In the last two examples our method allows to have chiral symmetry explicit during all stages of the calculation. The calculation for the model for the vector mesons agrees with our previous matching procedure[8] which was done by matching specific Green functions.

We have shown that with this procedure the non-analytical parts at one-loop level are also recovered in a few examples. We included both a two– and a three–point function in the simple model and a two–point function in the vector chiral case. These examples provide evidence that the “heavy” meson theory will also recover the correct terms non-analytical in the quark masses and the small momenta at the 1–loop level.

We discussed shortly the relevance of the nonzero width of the “heavy” particle.

Acknowledgments

PG acknowledges a grant form the Spanish Ministry for Education and Culture.
PG and PT thank the hospitality of the Theoretical Physics Department of Lund University, where part of this work has been carried out.

References

[1] N. Isgur and M. Wise, *Phys. Lett.* **B 232** (1989) 113.

[2] H. Georgi, *Phys. Lett.* **B 240** (1990) 447.

[3] M. Wise, *Phys. Rev.* **D 45** (1992) 2188;
G. Burdman and J. Donoghue, *Phys. Lett.* **B 280** (1992) 287.

[4] E. Jenkins and A. Manohar, *Phys. Lett.* **B 255** (1991) 558.

[5] E. Jenkins, A. Manohar and M. Wise, *Phys. Rev. Lett.* **75** (1995) 2272 [hep-ph/9506356].

[6] H. Davoudiasl and M. Wise, *Phys. Rev.* **D 53** (1996) 2523 [hep-ph/9509414];
J. Bijnens and P. Gosdzinsky, *Phys. Lett.* **B 388** (1996) 203 [hep-ph/9607462];
M. Booth et al., *Phys. Rev.* **D 55** (1997) 3092 [hep-ph/9610532].

[7] M. Luke and A. Manohar, *Phys. Lett.* **B 286** (1992) 348 [hep-ph/9205228], earlier work on chiral loops for arbitrary spins can be found in J. Gasser and A. Zepeda, *Nucl. Phys.* **B 174** (1980) 445 and J. Gasser, *Ann. Phys. (NY)* **136** (1981) 62.

[8] J. Bijnens, P. Gosdzinsky and P. Talavera, *Nucl. Phys.* **B 501** (1997) 495 [hep-ph/9704212].

[9] G. Ecker et al., *Nucl. Phys.* **B 321** (1989) 311.

[10] T. Mannel, W. Roberts and Z. Ryzak, *Nucl. Phys.* **B 368** (1992) 204.

[11] J. Gasser and H. Leutwyler, *Ann. Phys. (NY)* **158** (1984) 142.

[12] G. Ecker et al., *Phys. Lett.* **B 223** (1989) 425.

[13] J. Bijnens, P. Gosdzinsky and P. Talavera, in preparation.